Better Sharpe with Statistical Factor Models

Introduction

An optimal portfolio needs only an alpha score for each security together with a risk model (see Appendix A for details). A commercial risk model may not be available, whether for reasons of cost or the exotic nature of the universe. In these cases, a trailing series of returns for each security suffices to construct a risk model.

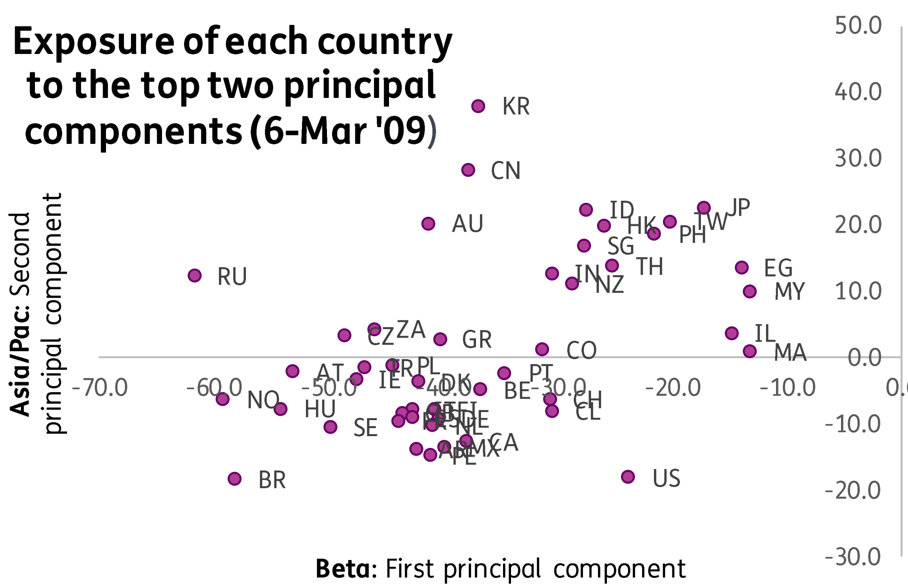
The problem with this widely-used approach is that one is implicitly assuming the future will be precisely like the recent past. While true of all risk models based on past returns, this driving-while-looking-at-the-rear-view-mirror problem can be mitigated by using statistical factor models. These models break down the dataset of past security returns into factors, considering only the most important ones. In this paper, we show how this can be done in a practical way.

Building the Risk Model

Statistical factor models use the first K principal components of the returns, the K directions capturing the most variance, as the risk factors. Risk, other than factor risk, for each security, is considered uncorrelated with that of any other security (i.e. idiosyncratic). Essentially one is expressing the view that the first K principal components of the returns represent signal while all the others are just random noise. For details, see Appendix B.

Friday, March 6, 2009

As an example, consider investing with the flow-based country model of Srimurthy, Shen and Smalbach [2018] at Friday, March 6, 2009. Here we are using the trailing 260 days of returns to invest in the 46 countries that belonged to the MSCI ACWI index at that time. Suppose also that we care about only the first two principal components.



The chart at right plots the second principal component against the first.

At March 6, 2009, the first principal component was beta, in that it had a 99.8% correlation with that factor. The second was Asia/Pacific, having, at the time, a 76% correlation with, the indicator variable that was one whenever a country falls in the Asia/Pacific region or zero otherwise. Basically, at that time, just before it bottomed, the market cared primarily about the beta of a country and, secondarily, whether that country was in Asia/Pacific.

Empirical results

We compared the returns of the market-neutral unit-risk optimal portfolio to that of naïve strategies that went long the top, and short the bottom, quintile of assets. We did this for many of EPFR's standard strategies.

The table at right shows annualized Sharpe ratios obtaining to both the naïve and optimal strategies. The Sharpe for the optimized back-test is shaded green if it is higher by 0.10 than that of the naïve Sharpe and shaded purple if it is lower by 0.10.

As you can see from the table at right, the Sharpe ratios for the optimal portfolio are invariably higher.

Naïve vs. Optimized backtests

	Annualized Sharpe	
Strategy	**Naïve**	**Optimized**
Flow-percentage country		
ACWI	0.55	0.61
EAFE	0.32	0.58
EM	0.43	0.33
Flow-percentage currency		
ACWI	0.96	1.11
G10	0.52	0.38
EM	0.43	0.67
Multi-asset strategy		
Multi-asset	0.51	1.27
Global 7 Equit	0.50	0.37
Fixed Income	0.72	1.03
Active/Passive country		
ACWI	0.45	0.66
EAFE	0.33	0.41
EM	0.12	0.38
Active/Passive sector		
EM sector	0.60	0.57
Japan sector	0.19	0.22
UK sector	0.43	0.36
Eurozone sector	0.47	0.69
US sector	0.45	0.70

Conclusion

Optimization is worthwhile, needs nothing more exotic than trailing returns, and tends to result in improved Sharpe ratios.

EPFR is happy to help implement optimization within your process.

Appendix

Appendix A: Finding the optimal market-neutral unit-risk portfolio

Suppose, for N securities, we have the following:

- α an $N \times 1$ matrix representing a score/alpha for each of those securities
- Ω an $N \times N$ matrix representing a risk model for these securities.

Let ω represent the optimal market-neutral unit-risk portfolio. Then $\langle\omega|\omega\rangle = \omega'\Omega\omega = 1$, where ω' denotes, the transpose of ω. Another constraint is that $\omega \cdot 1 = 0$. Further, $\omega \cdot \alpha$ is no less than $x \cdot \alpha$ for any other unit-risk portfolio x.

Said another way, $\omega \cdot \alpha = \omega'\alpha = \omega'\Omega\Omega^{-1}\alpha = \langle\omega|\Omega^{-1}\alpha\rangle$. The portfolio ω thus maximizes the covariance with $\Omega^{-1}\alpha$ amongst all unit-variance portfolios x so that $x \cdot 1 = 0$.

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Were it not for the market-neutral constraint, ω would have to be some linear multiple of $\Omega^{-1}\alpha$. Projecting $\Omega^{-1}\alpha$ onto the hyperplane perpendicular to $\Omega^{-1}1$, we see that ω must be some linear multiple of this projection. For any vector x, this projection is $x - (\langle x|\Omega^{-1}1\rangle / \langle \Omega^{-1}1|\Omega^{-1}1\rangle)\Omega^{-1}1 = x - \{x \cdot 1/(\Omega^{-1}1) \cdot 1\}\Omega^{-1}1$, where 1 is a vector with each entry equal to unity.

The precise scaling for ω is obtained by dividing this projection by its norm.

Appendix B: Statistical Factor Models

Let X be the $T \times N$ matrix of returns for N assets over T trailing time periods. Mean-adjust X, so the average return for each security, over these T periods is zero.

Perform a singular value decomposition of $X = UDV$ into the following three constituent pieces:

- U, a $T \times N$ matrix of risk factors or principal components. The columns of this matrix decrease in significance as one moves to the right
- V, an $N \times N$ matrix of unit determinant.
- D, an $N \times N$ diagonal matrix where the diagonal entries decrease as one moves to the right

Say one believes that only the first K columns of U contain signal and the remaining $T - K$ columns just represent noise. Then one can separate X into the following two pieces:

- $U_K \Delta_K$ where U_K is the $T \times K$ matrix representing the K most important columns of U and Δ_K, the first K rows of DV, is the $K \times N$ factor-exposure matrix of each security to each of the first K factors.
- $X - U_K \Delta_K$ is the $T \times N$ matrix representing the idiosyncratic, or non-factor-related, component of the returns of the N securities.

Now, the factor risk for each security is $\Delta_K'(U_K' U_K)\Delta_K/(N-1)$, an $N \times N$ matrix. Account for idiosyncratic risk by augmenting the i^{th} diagonal element of $\Delta_K'(U_K' U_K)\Delta_K/(N-1)$ by the dot product of the i^{th} column vector of $X - U_K \Delta_K$ by itself scaled by $N - 1$.

We have now arrived at our final risk model which can be used to construct the optimal market-neutral unit-risk portfolio.

References

Srimurthy, V. K., Shen, S., & Smalbach, M. (2018). Fund Flows as Country Allocator. The Journal of Alternative Investments, 21(3), 87-95.